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Mobile Radio Propogation:

Small Scale Fading and Multipath:

Small-scale fading, or simply fading is used to describe the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance, so that large-scale path loss effects may be ignored.

Factors Influencing Small-Scale Fading:

1. multipath propogation

   The random phase and amplitudes of the different multipath components cause fluctuations in signal strength, thereby inducing small-scale fading, signal distortion, or both

2. Speed of the mobile

   The relative motion between the base station and the mobile results in random frequency modulation due to different Doppler shifts

3. Speed of the surrounding environment

4. The ~~transmission~~ bandwidth of the signal

   If the ~~transmitted~~ radio signal bandwidth

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is greater than the bandwidth of the multipath channel, the received signal will be distorted

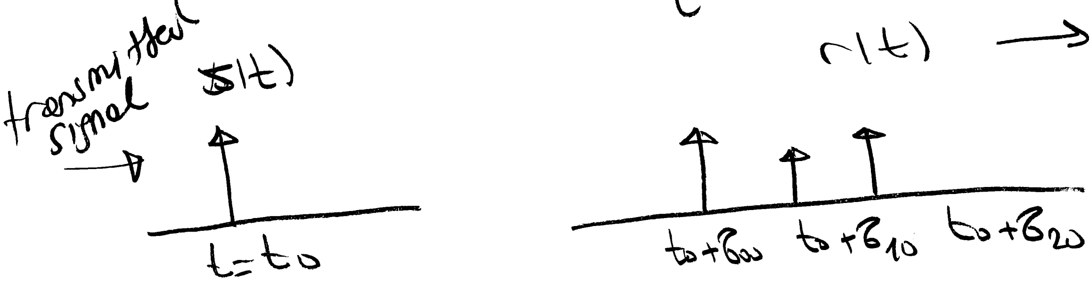
Impulse Response Model of a Multipath Channel

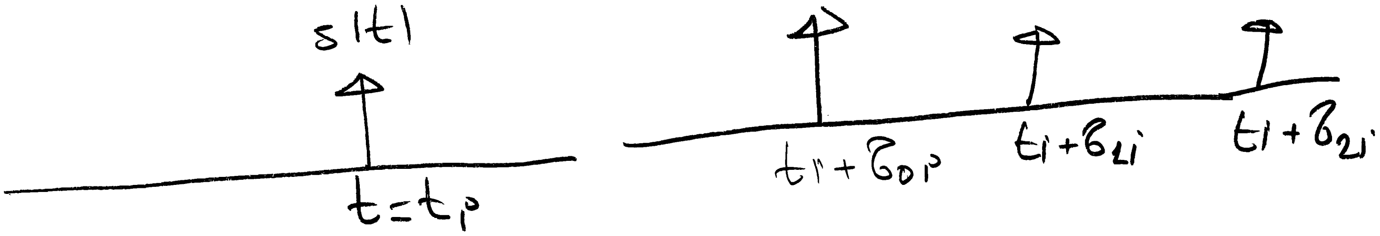
$s(t) \rightarrow$ Bandpass signal $\qquad f_c \rightarrow$ carrier frequency

$s_e(t) \rightarrow$ Lowpass signal

$$s(t) = \mathrm{Re}\left\{ s_e(t)\, e^{j2\pi f_c t} \right\}$$

$r(t) \longrightarrow$ received signal





(3) The received bandpass signal can be written as

$$r(t) = \sum_n \alpha_n(t)\, s(t - \tau_n(t))$$

where $\alpha_n(t)$ is the attenuation factor for the signal received on the nth path and $\tau_n(t)$ is the propagation delay for the nth path.

Since $s(t) = \mathrm{Re}\{ s_l(t) e^{j2\pi f_c t} \}$

then $$r(t) = \sum_n \alpha_n(t)\, \mathrm{Re}\left\{ s_l(t - \tau_n(t))\, e^{j2\pi f_c (t - \tau_n(t))} \right\}$$

$$r(t) = \mathrm{Re}\left\{ \left(\sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, s_l(t - \tau_n(t)) \right) e^{j2\pi f_c t} \right\}$$

In the absence of noise the equivalent lowpass received signal is

$$r_l(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, s_l(t - \tau_n(t))$$

Ex:

$$r(t) = \alpha_1 s(t - \tau_1) + \alpha_2 s(t - \tau_2) + \alpha_3 s(t - \tau_3)$$

$r(t) \to$ received signal $\quad s(t) \to$ transmitted signal

Find impulse response of the channel

(4) Sln:

$$r(t) = \alpha_1 s(t-\tau_1) + \alpha_2 s(t-\tau_2) + \alpha_3 s(t-\tau_3)$$

Remember the convolution

$$s(t) \otimes h(t) = \int s(\tau) h(t-\tau) d\tau$$

OR $$s(t) \otimes h(t) = \int h(\tau) s(t-\tau) d\tau$$

and impulse function $\delta(t)$ has the following properties

$$\int \delta(t-t_0) x(t) dt = x(t_0)$$

$$\int \delta(t-t_0) x(t-t_1) dt = x(t_0 - t_1)$$

$$\delta(t-t_0) x(t) = x(t_0) \delta(t-t_0)$$

$$\int \delta(t) dt = 1$$

Hence;

$$r(t) = \alpha_1 s(t-\tau_1) + \alpha_2 s(t-\tau_2) + \alpha_3 s(t-\tau_3)$$

$$= \alpha_1 \int \delta(\tau-\tau_1) s(t-\tau) d\tau + \alpha_2 \int \delta(\tau-\tau_2) s(t-\tau) d\tau$$

$$+ \alpha_3 \int \delta(\tau-\tau_1) s(t-\tau) d\tau$$

$$r(t) = \int \underbrace{\left[\alpha_1 \delta(\tau-\tau_1) + \alpha_2 \delta(\tau-\tau_2) + \alpha_3 \delta(\tau-\tau_3)\right]}_{h(\tau)} s(t-\tau) d\tau$$

⑤ Thus

$$h(\tau) = \alpha_1\, \delta(\tau-\tau_1) + \alpha_2\, \delta(\tau-\tau_2) + \alpha_3\, \delta(\tau-\tau_3)$$

$h(\tau) \rightarrow$ Impulse response of the channel.

α_i & τ_i $\rightarrow$ attenuation and delay parameters

α_i & τ_i $\rightarrow$ depends on t_i ; i.e, time instant

$$h(\tau) = \alpha_1\, \delta(\tau-\tau_1) + \alpha_2\, \delta(\tau-\tau_2) + \alpha_3\, \delta(\tau-\tau_3)$$

$$h(t) = \alpha_1\, \delta(t-\tau_1) + \alpha_2\, \delta(t-\tau_2) + \alpha_3\, \delta(t-\tau_3)$$

$\downarrow$

This channel is time invariant

Since its impulse response attenuation and delay parameters does not change in time they are all fixed

i.e., $h(t_0) = \alpha_1 \delta(t_0-\tau_1) + \alpha_2 \delta(t_0-\tau_2) + \alpha_3 \delta(t_0-\tau_3)$

$$h(t_1) = \alpha_1 \delta(t_1-\tau_1) + \alpha_2 \delta(t_1-\tau_2) + \alpha_3 \delta(t_1-\tau_3)$$

For a time varying channel attenuation, delay and other parameters change in time.

Ex:

$$h(t) = \alpha_1(t)\, \delta(t-\tau_1(t)) + \alpha_2(t)\, \delta(t-\tau_2(t)) + \alpha_3(t)\, \delta(t-\tau_3(t))$$

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constant value for t; constant value; const. value; const. value; const. value

$$h(t) = \alpha_1(t)\,\delta(t-\tau_1(t)) + \alpha_2(t)\,\delta(t-\tau_2(t)) + \alpha_3(t)\,\delta(t-\tau_3(t))$$

shows time dependency; "change parameter t to τ", values are kept the same

$$h(\tau,t) = \alpha_1(t)\,\delta(\tau-\tau_1(t)) + \alpha_2(t)\,\delta(\tau-\tau_2(t)) + \alpha_3(t)\,\delta(\tau-\tau_3(t))$$

OR

$$h(t,\tau) = \alpha_1(t)\,\delta(\tau-\tau_1(t)) + \alpha_2(t)\,\delta(\tau-\tau_2(t))$$

then

$$r(t) = \int_{-\infty}^{\infty} h(t,\tau)\, s(t-\tau)\, d\tau$$

$h(t,\tau) \to$ time varying channel impulse response.

$$r(t) = \int_{\infty}^{\infty} h(t,\tau)\, s(t-\tau)\, d\tau$$

← shown as

$$= h(t,\tau) * s(t)$$

Note that the above definition is different than the classical convolution definition

6a) OR

$$h(t,t') = \alpha_1(t)\,\delta(t' - \tau_1(t)) + \alpha_2(t)\,\delta(t' - \tau_2(t)) + \alpha_3(t)\,\delta(t' - \tau_3(t))$$

$t = t'$

t, t' both show time, (the same time)

$\alpha_i(t), \tau_i(t) \rightarrow$ real values depending on time t

$$h(t,\tau) = \alpha_1(t)\,\delta(\tau - \tau_1(t)) + \alpha_2(t)\,\delta(\tau - \tau_2(t)) + \alpha_3(t)\,\delta(\tau - \tau_3(t))$$

$$r(t) = \int_{-\infty}^{\infty} h(t,\tau)\, s(t-\tau)\, d\tau$$

$$r(t) = h(t,t') \circledast s(t)$$

$h(t,\tau)$ or $h(t,t')$ can both be used to denote impulse response of the wireless channel.

(7) Lowpass Impulse Response (General Expression)

If $r_e(t) \rightarrow$ lowpass received signal
drop subscript e for simplicity
from now on
i.e., $r(t) \rightarrow$ lowpass received signal

$$r(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t) + \phi_i(t,\tau)}\, s(t-\tau_n(t))$$

The impulse response becomes as

$$h(t,\tau) = \sum_{n,N} \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t) + \phi_i(t,\tau)}\, \delta(\tau-\tau_n(t))$$

(A) $\alpha_n(t) \rightarrow$ attenuation and delays
$\tau_n(t) \rightarrow$ for the nth multipath component at time t

$\phi_i(t,\tau) \rightarrow$ additional phase shift encountered at channel

$$h(t,\tau) = \sum_{n=0}^{N-1} \alpha_n(t,\tau)\, e^{-j\theta_n(t,\tau)}\, \delta(\tau-\tau_n(t))$$

$\alpha_n(t,\tau) \rightarrow$ attenuation at time instant t and the delay τ (nth path) same as (A)
Just more parameters are used

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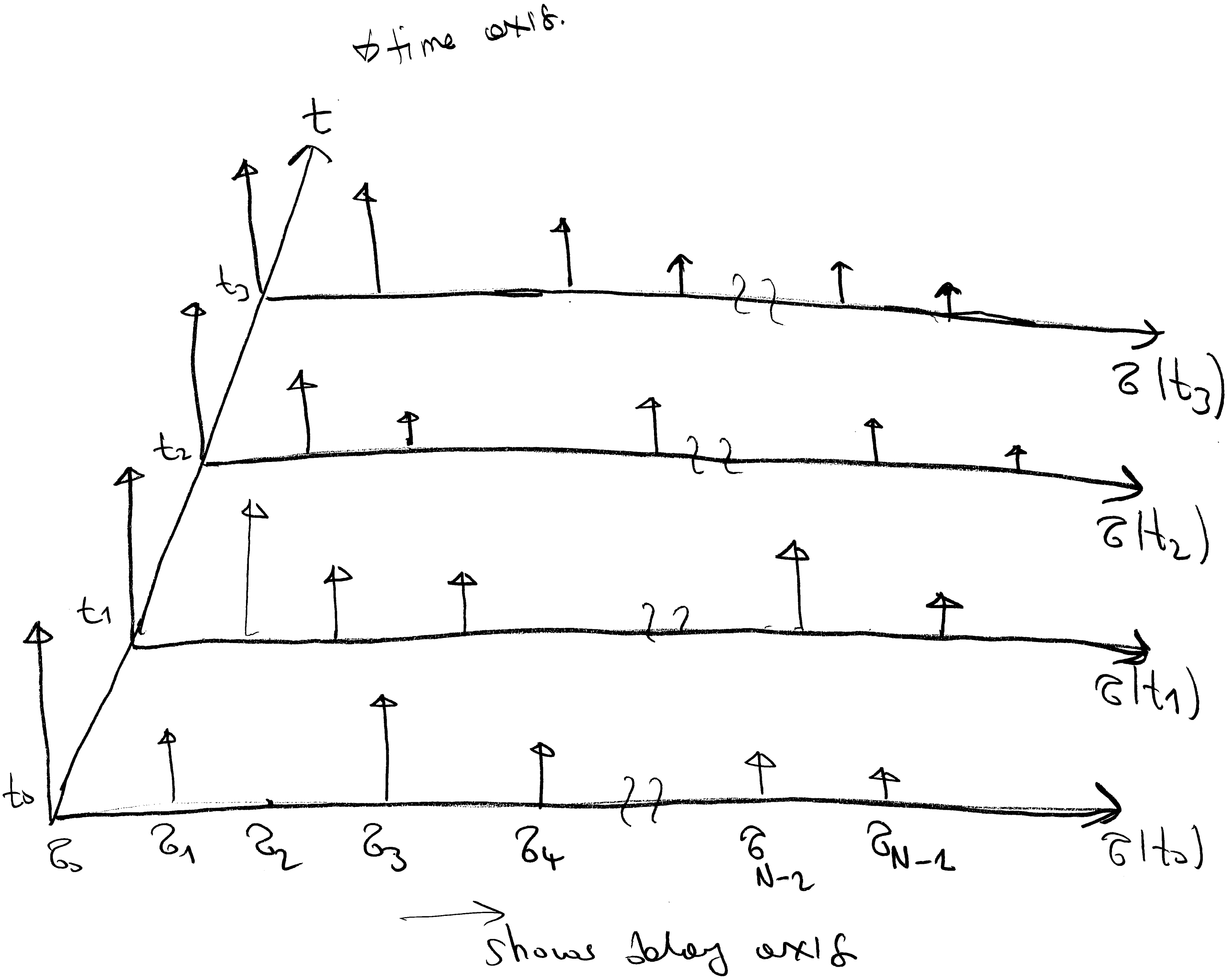
Graphical Illustration of the Impulse Response of Time Varying Channel



The impulse response of the channel for time instant t_0 is different than the impulse response of the channel for time instant t_1

In general $h(t_i, \tau) \neq h(t_j, \tau)$

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If the channel impulse response is assumed to be time invariant, or is at least wide sense stationary over a small-scale time, then the channel impulse response may be simplified of

$$h(\tau) = \sum_{i,N} \alpha_i \exp(-j\theta_i)\,\delta(\tau-\tau_i)$$

Relationship Between Bandwidth and Received Power:

Consider the transmitted RF signal of the form $x(t) = \mathrm{Re}\{p(t)\exp(j2\pi f_c t)\}$

$p(t) \rightarrow$ repetitive baseband pulse train with very narrow pulse width T_{bb} and repetition period T_{REP} which is much greater than the maximum measured excess delay τ_{max} in the channel.

Let $p(t) = 2\sqrt{\tau_{max}/T_{bb}}$ for $0 \le t \le T_{bb}$

The low pass channel output $r(t)$ closely approximates the impulse response $h_b(t)$ and is given by

(10)

$$r(t) = \frac{1}{2} \sum_{i=0}^{N-1} a_i \left(\exp(-j\theta_i)\right) p(t-\tau_i)$$

$$= \sum_{i=0}^{N-1} a_i \exp(-j\theta_i) \sqrt{\frac{\tau_{max}}{T_{bb}}} \operatorname{rect}\left(t - \frac{T_{bb}}{2} - \tau_i\right)$$

$|r(t_0)|^2 \rightarrow$ instantaneous multipath power delay profile

$$|r(t_0)|^2 = \frac{1}{\tau_{max}} \int_0^{\tau_{max}} r(t) r^*(t)\, dt$$

$$|r(t_0)|^2 = \frac{1}{\tau_{max}} \int_0^{\tau_{max}} r(t) r^*(t)\, dt$$

$$= \frac{1}{\tau_{max}} \int_0^{\tau_{max}} \frac{1}{4} \operatorname{Re}\left\{ \sum_{j,N} \sum_{i,N} a_j(t_0) a_i(t_0) p(t-\tau_j) p(t-\tau_i) \exp\left(-j(\theta_j - \theta_i)\right)\right\} dt$$

for all $j \neq i$

$$|r(t_0)|^2 = \frac{1}{\tau_{max}} \int_0^{\tau_{max}} \frac{1}{4} \left(\sum_{k,N} a_k^2(t_0) p^2(t-\tau_k) \right) dt$$

$$= \frac{1}{\tau_{max}} \sum_{k,N} a_k^2(t_0) \int_0^{\tau_{max}} \left\{ \sqrt{\frac{\tau_{max}}{T_{bb}}} \operatorname{rect}\left(t - \frac{T_{bb}}{2} - \tau_k\right) \right\}^2 dt$$

$$= \sum_{k,N} a_k^2(t_0)$$